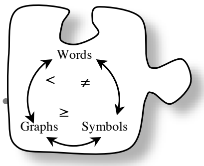
5.2.4 How can I find all solutions?



Solving One Variable Inequalities

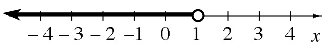
In this lesson, you will work with your team to develop and describe a process for solving linear inequalities. As you work, use the following questions to focus your discussion.

What is a solution?

What do all of the solutions have in common?

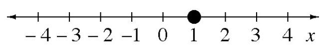
What is the greatest solution? What is the smallest solution?

5-72. Jerry and Ken were working on solving the inequality $3x-1 \le 2x$. They found the boundary point and Ken made the number line graph shown at right.



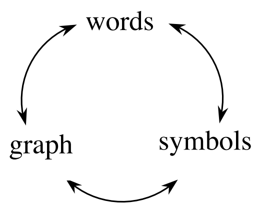
Jerry noticed a problem. *"Doesn't the line at the bottom of the* $\le$ *symbol mean the equal part? That means that* $x=1$ *is also a solution. How could we show that?"*

"Hmmm," Jerry said. *"Well, the solution* $x=1$ *would look like this on a number line. Is there a way that we can combine the two number lines?"*



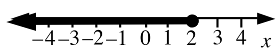
Discuss this idea with your team and be prepared to share your ideas with the class.

5-73. The diagram at right shows three possible ways to represent inequality statements. Review the meanings of the inequality symbols >, <, ≥, and ≤ with your team. Then, generate the two missing representations from each inequality described in parts (a) through (c) below.



a. $x < -1\frac{1}{2}$

b. x is greater than or equal to two.

c. 

5-74. WHEN IS THE BOUNDARY POINT INCLUDED?

Represent the solution for each of the variables described below as an inequality on a number line graph and with symbols.

a. The speed limit on certain freeways is 65 miles per hour. Let x represent any speed that could get a speeding ticket.

b. You brought $10 to the mall. Let y represent any amount of money you can spend.

c. In order to ride your favorite roller coaster, you must be at least five feet tall but less than seven feet tall. Let h represent any height that is allowed to ride the roller coaster.

5-75. Jordyn, Geri, and Morgan are going to have a kite-flying contest. Jordyn and Geri each have one roll of kite string. Together they also have 90 yards of extra string. Morgan has three rolls of kite string plus 10 yards of extra string. All of the rolls of string are the same length. The girls want to see who can fly their kite the highest.

a. Since Jordyn and Geri have fewer rolls of kite string, they decide to tie their string together so their kite can fly higher. Write at least two expressions to show how much kite string Jordyn and Geri have. Let x represent the number of yards of string on one roll.

b. Write an expression to show how much kite string Morgan has. x is the number of yards of string on one roll.

c. How long does a roll of string have to be for Jordyn and Geri to be able to fly their kite higher than Morgan's kite? Show your answer as an inequality and on a number line.

d. How long does a roll of string have to be for Morgan to be able to fly her kite higher than Jordyn and Geri's kite? Show your answer as an inequality and on a number line.

e. What length would the roll of string have to be for the girls' kites to fly at the same height?

5-76. **Additional Challenge:** Travis loves trains! Today he is beginning a train ride from Madison, Wisconsin all the way to Seattle, Washington.

Shortly after the train left the station in Madison, Travis fell asleep. When he woke up, it was dark outside and he had no idea how long he had been asleep. A fellow passenger told him they had already passed La Crosse, which is 135 miles from Madison. If the train travels at an average speed of 50 miles per hour, at least how long has Travis been asleep? Work with your team to represent this problem with an inequality and to solve it. Be prepared to share your ideas with the class.

5-77. LEARNING LOG

Work with your team to describe each step of your process for finding boundary points and deciding what part of the number line to shade. Then write down each step in your Learning Log. Be sure to illustrate your ideas with examples. Title this entry “Finding Boundary Points” and label it with today’s date.

Review & Preview

5-78. Solve each of the following inequalities. Represent the solutions algebraically (with symbols) and graphically (on a number line).

a. $3x-4<3-2x$

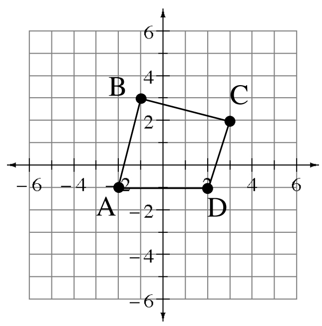
b. $\frac{4}{5}x \geq 7$

5-79. Kindra would like to have more than \$1500 in her savings account.

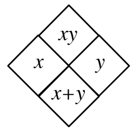
a. If she starts with \$61 in her savings account, write an inequality to show how much she wants to have.

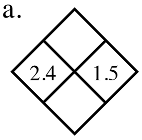
b. How much does Kindra need to save?

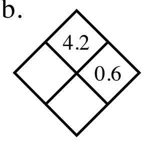
5-80. Reflect quadrilateral ABCD across the line $y=-2$. Write the new coordinate points.

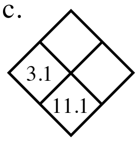


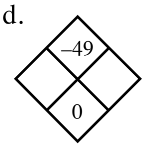
5-81. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



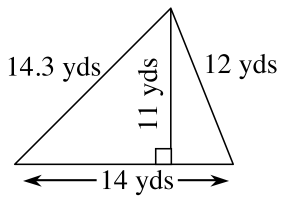
a.


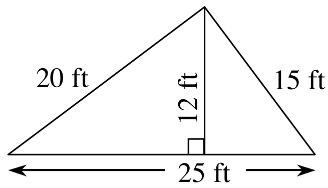
b.


c.


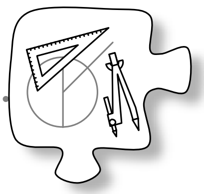
d.


5-82. Find the perimeter and area of each triangle below.

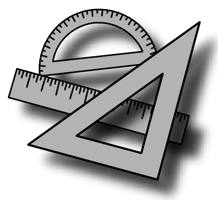
a.


b.


5.3.1 What can I build?



Introduction to Constructions



Architects and drafters make careful drawings before buildings are built. The drawings represent how the building should be constructed. They show the thickness of the walls and where doors, windows, and pipes will be. To make these precise diagrams, professionals often use special computer software. However, some architects prefer to draw the diagrams by hand. They use special drawing tools that look something like those in the drawing at right, including a right triangle tool and a device for measuring angles called a protractor.

Building plans are just one example of a precise geometric drawing. Mathematicians have used these basic tools to draw accurate diagrams for other reasons. The process of **constructing**, or building, shapes that meet different guidelines can help you understand many of the specific characteristics of those shapes.

In this lesson, you will be introduced to some ideas of mathematical construction and will construct your own shapes. As you work today, use the questions below to start mathematical discussions:

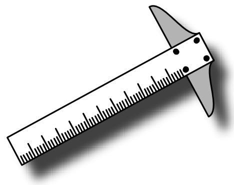
What is the relationship?

How can we describe it?

How do we know for sure?

5-83. TRANSLATING THE LINE SEGMENT

One of the most basic drafting tools used is called a T-square, a long straightedge that has a "T" at one end. This tool slides along an edge of a table to allow a person to draw several lines that go in exactly the same direction, as if they were translated on the page. An example of a T-square is shown at right.



Using tracing paper provided by your teacher, explore how a T-square works.

- Trace the line segment AB on the Lesson 5.3.1A Resource Page on tracing paper along with the "grid marker," an ∟ shape that can help you keep the tracing paper lined up with the grid as you move it.
- Use the tracing paper to translate the segment AB on the tracing paper vertically (↕) and horizontally (↔) on the graph any distance you choose. Be careful not to rotate the tracing paper or the resource page! One way to make sure that they stay lined up is to make sure that the ∟ shape matches up with two intersecting gridlines.
- How did the new translated segment compare to the original?
- Translate the segment to different positions on the grid. Pay attention to how the translated segment and the original segment compare.

Based on the translations you have made, what is the relationship between two line segments when one is a translation of the other? On the resource page, sketch and describe the **general** relationship you see.

5-84. ROTATING THE LINE SEGMENT

You have seen how line segments are related when one is a translation of the other. How does the relationship change if the segment is rotated? Trace segment *CD* and the grid marker from the Lesson 5.3.1A Resource Page to begin investigating.

- With the traced segment matched up on top of the original line segment, rotate the line segment 90° counter-clockwise (↺) about point *C*. Your grid marker will look like ⌟ when you have rotated a complete 90°. Pay attention to how the rotated segments compare to the original line. Sketch your line segment on the graph and label the endpoint *E*.
- Use a straightedge to extend segments *CD* and *CE* by starting at point C and moving out along each segment. Place an arrow at the end of each segment. The 90° angle formed by this rotation is a special angle called a **right angle**.
- Line your traced segment up with segment *CD* again. Choose another point on the segment and rotate the segment 90° clockwise or counter-clockwise about that point. Sketch where the new segment lands.

a. What relationships do you notice between the rotated line segments and the original segment *CD*?

b. When lines meet and form 90° angles they are called **perpendicular** lines. Are the rotated line segments you drew **perpendicular** to segment *CD*? Explain why or why not.

c. How many total times would you need to rotate your line segment by 90° around the endpoint so that the rotated segment forms a straight line segment with the original segment? The new segment should be twice the original length. How many degrees of rotation is that?

d. How many times would you rotate the segment by 90° to complete a full turn and end up back where you started?

5-85. MEASURING ANGLES

Different angles are created when lines are rotated by different amounts around a point of intersection.

a. On the Lesson 5.3.1B Resource Page, locate the segment with endpoints at points *A* and *B*.

- Trace segment *AB* on tracing paper and rotate it more than 90° (a $\frac{1}{4}$ turn) and less than 180° (a $\frac{1}{2}$ turn) about point *A*.
- Copy this segment onto the resource page using a straightedge.
- Label the new endpoint as point *C*.

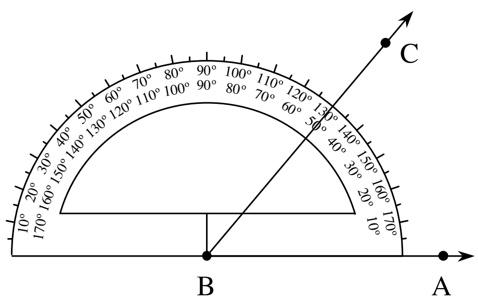
Estimate how many degrees you have rotated the line segment. What kind of angle is this?

b. Find segment *GH* on your resource page.

- Use your tracing paper to rotate the segment around point *G* less than 90° (a $\frac{1}{4}$ turn).
- Copy the rotated segment onto the resource page using a straightedge.
- Label the new endpoint as point *K*.

Estimate how many degrees you have rotated the line segment. Be prepared to explain your thinking. What kind of angle is this?

5-86. A **protractor** is a tool for measuring angles. With a protractor you can measure the number of degrees of rotation that create an angle. Cliff used the protractor pictured here to measure the acute angle he created.



a. What is the measure of his angle? Make sure that your answer makes sense based on the size of the angle.

b. Obtain a protractor from your teacher and measure each of the angles you created in problem 5-85. How close were your estimates to the actual measurements?

c. Draw a new angle on your paper using a straightedge. Determine if it is obtuse, acute, or right, and estimate its measure in degrees. Then use your protractor to measure the angle. How close was your estimate? Use the Math Notes box at the end of this lesson if you need help with the names of the angle types.

5-87. Architects and contractors often need to create angles that measure 45° or 60° as they build. Their measurements must be accurate in order for the buildings they design to fit together when they are built.

a. On your paper, create a 45° angle by following the steps below.

- Use a straightedge to draw a line at least 4 inches long.
- Mark a point on one end of the line and label it *X*.
- Position the center point of your protractor on the point you marked *X* and the zero degree mark on the line.
- From the line, read around the edge of the protractor to find the 45° mark. Make a point at 45°.
- Draw a line connecting point *X* to your new point to create the angle with a measure of 45°.

Label the angle with the measure 45°. Is your drawing **reasonable**?

b. Draw three new angles with the measures below. Label each one with its measure.

i. 30° *ii.* 75° *iii.* 135°

5-88. **Additional Challenge:** If you make four 90° rotations in the same direction, one after the other around the same point, you will complete a full circular angle.

a. How many 45° rotations does it take to make a circular angle?

b. Suppose you want to divide a circular angle into 3 angles of equal measure. What would the degree measure of each angle be? Draw and label a figure using a protractor.

c. Suppose you want to divide a circular angle into 5 angles of equal measure. What would the degree measure of each angle be? Draw and label a figure using a protractor.

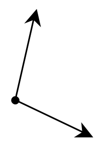
d. Suppose you want to divide a circular angle into 24 angles of equal measure. What would the degree of each angle be?

MATH NOTES

METHODS AND MEANINGS

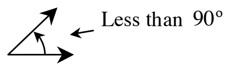
Angles and Their Measures

An **angle** is formed by two rays that start at a single point (called an **endpoint**), as shown in the diagram at right. A **ray** is part of a line that starts at an endpoint and extends without stopping in one direction. The **measure** of an angle is how much you rotate the starting ray to get to the ray that forms the other side of the angle. In this course, angles will be measured in degrees.

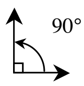


When trying to describe shapes, it is convenient to classify them by types of angles. This course will use the following terms to refer to angles:

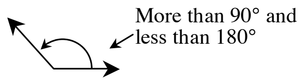
Acute: Any angle with measure *between* (but not including) 0° and 90°.



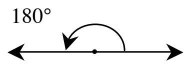
Right: Any angle that measures 90°.



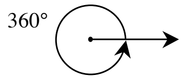
Obtuse: Any angle with measure *between* (but not including) 90° and 180°.



Straight: Straight angles have a measure of 180° and the two rays of the angle form a straight line.



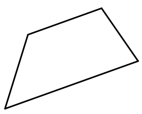
Circular: Any angle that measures 360°.



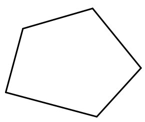
Review & Preview

5-89. Look at the shapes below. Determine if they include a parallel set of line segments, a perpendicular set of line segments, both, or neither one.

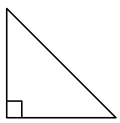
a. trapezoid



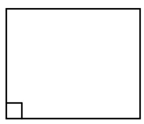
b.



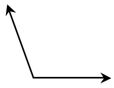
c.

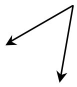


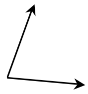
d. rectangle

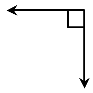


5-90. Decide if these angles are acute, obtuse, or right angles. All diagrams are drawn to scale.

a. 

b. 

c. 

d. 

5-91. Edwin's friends guessed how many jelly beans were in a jar at his birthday party. Here are their guesses: 75, 80, 95, 92, 100, 72, 71, 60, 65, 88, 60.

a. Make a stem-and-leaf plot to display the data.

b. Make a box-and-whisker plot to display the data.

c. Which data display most clearly shows the median of the data? What is the median?

d. Based on the box-and-whisker plot, estimate what percent of students guessed more than 65 jellybeans.

e. Use the stem-and-leaf plot to determine the actual percent of students who guessed more than 65 jellybeans.

5-92. Copy each problem, then find the sum, difference, product, or quotient. Remember to show all your steps.

a. $23.6 + 12$ b. $16.5 + 52.43$

c. $46.21 - 31.2$ d. $27.5 - 13.11$

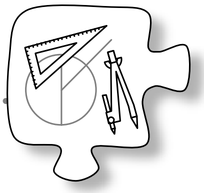
e. $4.5(6)$ f. $55 \div 2$

5-93. Mr. Crow, the head groundskeeper at High Tech Middle School, mows the lawn along the side of the gym. The lawn is rectangular, and the length is 5 feet more than twice the width. The perimeter of the lawn is 250 feet.

a. Use the 5-D Process to find the dimensions of the lawn.

b. Use the dimensions you calculated in part (a) to find the area of the lawn.

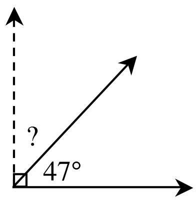
5.3.2 What is the relationship?



Compass Constructions

A compass is a simple tool that allows circles to be drawn quickly and accurately. Using this simple tool along with a ruler allows one to make precise drawings. In this lesson you will use a compass and ruler to build shapes, construct perpendicular lines, and partition segments.

5-94. Benjamin was drawing angles with a broken protractor and the largest angle he could measure was a 47° angle. Benjamin wanted to draw a 90° angle as well as a 180° angle.



a. Suppose Benjamin started by drawing a 47° angle. What angle would he need to measure to complete his right angle?

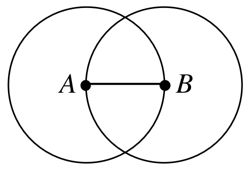
b. When the sum of the measures of two angles is 90°, they are called **complementary angles**. What angle would be complementary to an 83° angle? Sketch a right angle showing this relationship.

c. Angles that sum to make a 180° angle are called **supplementary angles**. What angle would you need to add to the angles below to create a 180° angle?

i. 32° *ii*. 90° *iii*. 75°

5-95. DESIGNS WITH CIRCLES

Celeste was playing with her compass and created the figure at right. When a compass and/or a straightedge is used to create a figure, the process is called **constructing**. Use a straightedge to draw a horizontal segment on your paper that is two inches long, label the endpoints *A* and *B*, then follow the directions below to recreate her **construction**.



- Use your compass to construct a circle that has a center at point *A* and passes through point *B*.
- Construct a second circle that has a center at point *B* and passes through point *A*. Label the points where the circles **intersect** (cross) as points *C* and *D*.

Problem continues on next page. →

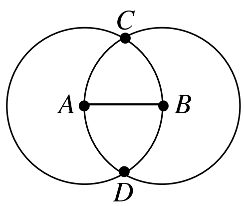
5-95. *Problem continued from previous page.*

- Use a straightedge to draw segments from the center of each circle to point *C* and point *D*. Each of these line segments is a **radius** of the circle. A **radius** measures the distance from the center of the circle to any point on the circle itself.

a. How are the four segments related? Is there another pair of points in the figure that create a segment that is the same as the ones you drew?

b. What shapes do you see in the construction? Describe them and **justify** your descriptions.

5-96. In part (a) of problem 5-95 you found that the segment that connects point *A* to point *B* is also a radius. Now connect point *C* to point *D*.



a. How are segments AB and CD related? Use your protractor to measure the angles where line segment CD crosses segment AB to confirm the relationships.

b. The word **bisect** means to cut into two equal pieces. Use a ruler to check that the segment connecting *C* and *D* goes through the **midpoint** (exact middle) of segment *AB*. Segment *CD* is called a **perpendicular bisector** of the segment connecting point *A* and point *B*.

c. On a new piece of paper, use a straight edge to draw a line segment. Label the endpoints E and F. Use the construction process you used to construct segment CD (problems 5-95 and 5-96) to construct a perpendicular bisector for segment EF.

5-97. SPECIAL TRIANGLES

Two of the triangles you created on the intersecting circles in problem 5-95 have special properties.

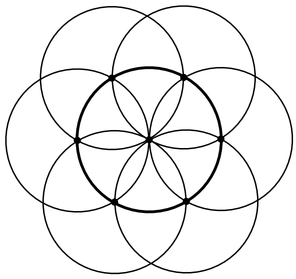
a. Based on your work in the previous problems, how are the lengths of the sides of triangle *ABC* related?

b. Examine triangle *ABD*. How is it related to triangle *ABC*?

c. Use your protractor to measure each of the angles in triangle *ABC*. What do you notice about the angles?

Triangles like triangle *ABC* and *ABD* are called **equilateral** (all sides equal) and **equiangular** (all angles equal) because of the special relationships between their sides and angles.

5-98. **Additional Challenge:** FLOWER PETALS



Using the idea of overlapping circles, Samantha constructed the picture at right. Follow the directions below to construct her pattern on your paper. Then answer the questions that follow.

- In the middle of a large blank piece of paper, construct a circle. Be sure to mark the center of the circle. This point will be the center of the pattern.
- Pick and label a point (*A*) on the circle. Then use your compass to construct a circle with the same radius as the original circle, but with its center at point *A*. This new circle should intersect your original circle twice. Call these points *B* and *C*.
- Now draw new circles with centers at points *B* and *C* so that each circle has the same radius as the original circle. Each circle should intersect the original circle in two new points. Label these points *D* and *E*.
- Finally, draw two new circles with centers *D* and *E* so that they have the same radius as the original circle. They should intersect the original circle at the same point. Label this point *F*.

a. Find a "flower petal" pattern in your diagram (you may see more than one). You may want to color the petals so that they are easy to find.

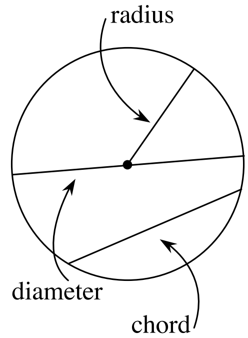
b. If you connect the intersection points (*A*, *B*, *C*, …) in order around the original circle, what shape appears? Describe it.

METHODS AND MEANINGS

MATH NOTES

Circle Vocabulary

The **radius** of a circle is a line segment from its center to any point on the circle. The term is also used for the length of these segments. More than one radius are called **radii**.



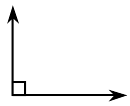
A **chord** of a circle is a line segment joining any two points on a circle.

A **diameter** of a circle is a chord that goes through its center. The term is also used for the length of these chords. The length of a diameter is twice the length of a radius.

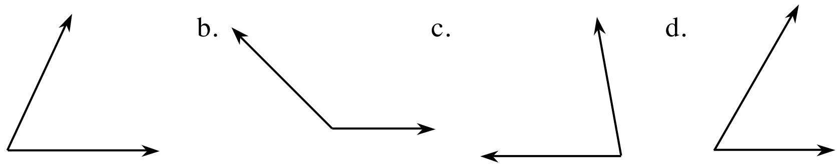
The **circumference** of a circle is its perimeter, or the "distance around" the circle.

Review & Preview

5-99. The angle at right measures 90°. Estimate the measure (in degrees) of each angle below. Note: The angles are drawn to scale.



a. b. c. d.



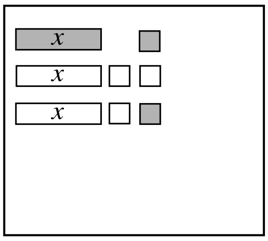
e. If the angle in part (a) measures 65°, what would be the measure of its supplementary angle? **Justify** your answer.

5-100. What number do you get with this number "magic trick"? Use at least two different original numbers for step #1 to confirm your solution.

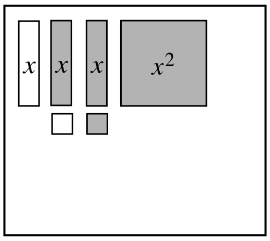
1. Think of a number.
2. Add the next higher number.
3. Add nine.
4. Divide by two.
5. Subtract your original number.

5-101. Write the expression as shown on the expression mats, then simplify by making zeros and combining like terms.

a.

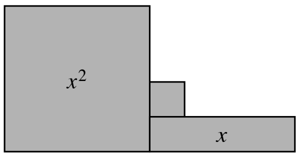


b.



5-102. Draw a triangle. Choose one side as the base, and label it 12 inches. Draw the height perpendicular to the base. If the area of the triangle is 138 square inches, what is the height? You may find it helpful to use the 5-D Process.

5-103. Look at the algebra tile shape at right.



a. Write an algebraic expression for the perimeter of the shape in two ways, first by finding the length of each of the sides and adding them all together and then by writing an equivalent, simplified expression.

b. Write an algebraic expression for the area of the shape.

5-104. The average annual rainfall in Tucson, Arizona is 12 inches. Between January and April, 2.4 inches of rain fell. What percentage of the annual rainfall fell after April (May through December)? You may want to draw a diagram to organize information.

5.3.3 What is the relationship?

Circumference and Diameter Ratios

The ability to measure objects without standard measuring tools is often very convenient. Have you ever seen anyone estimate the time until the sun sets by extending their arms and seeing how many fists there are from the horizon to the sun? Others use the distance from the tip of their thumb to their knuckle as an approximate inch. Today you will use your own foot as a unit of measure to look at another mathematical relationship.

5-105. WALKING CIRCLES

Manny was walking around the edge of the circle painted on one of the basketball courts on the playground. He was carefully placing his feet heel-to-toe and counting as he walked. After he went all the way around the circle, he started to walk along the line across the middle of the circle.

His friend Jose was watching Manny walk this way and thought it was unusual. Jose asked, "*Why are you walking like that?*"

"*I found a pattern in the number of steps I have to take around the circle and the number of steps across the circle. I'm testing to see if the relationship is true on different circles,*" Manny answered.

What relationship did Manny find? Today you and your team will walk around and across circles to see if you can find a pattern. You will need a Lesson 5.3.3A Resource Page to record your measurements (available at www.cpm.org/students).

a. Your teacher will direct your team to a circle to measure. Measure the distance around the circle, called the **circumference**, using Manny's walking method. How many heel-to-toe steps does it take to trace the full circle? Record your result on the resource page.

b. Measure the **diameter**, or distance across the circle through the center, once again using heel-to-toe units. Make sure you carefully count your steps and record your measurement.

c. When each person in your team has made measurements using their own heel-to-toe units, look at your results. As a team, discuss the patterns you see. What relationship did Manny find? Write your observations and be prepared to explain your ideas.

5-106. You may have noticed that the circumference measurement, C, was always larger than the diameter measurement, d, for the circles you measured. Could you determine how much larger?

a. Divide each circumference measurement by the diameter measurement. Record your results in the last column of your team chart, and label the column "$\frac{C}{d}$."

b. What relationship do you see in the results? Discuss with your team and then write a sentence summarizing the relationship between circumference and diameter.

5-107. Looking at large amounts of data can help to confirm patterns that you identify in smaller data sets. To share your team data with the class, find the mean of your team's data in each column of the chart on your resource page and add your results to the class chart. Record the class results on your own resource page.

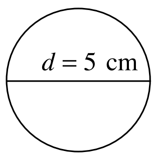
a. Are the class results similar to what your team found when it measured? Explain any differences you see, and what may have caused them.

b. Do the class results confirm the patterns you saw in problems 5-105 and 5-106? Describe any new patterns that you see and, if you are revising the patterns you found, explain why you are doing so.

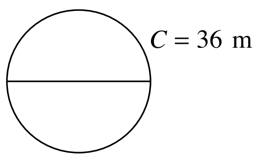
c. With your team, go back to the sentence you wrote in part (b) of problem 5-106, summarizing the relationship between circumference and diameter. Add to or change your summary as needed.

5-108. Use the relationships you have found to approximate the missing circumference and diameter measurements.

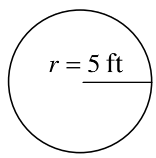
a. circumference = ?



b. diameter = ?



c. circumference =?
diameter = ?



5-109. Mathematicians have studied circles and the challenge of measuring them accurately for many years. Through their studies, mathematicians identified the ratio between the circumference and diameter of a circle as a special number. No matter how large or small a circle is, and no matter what units of measure are used, the circumference divided by the diameter is always equal to a little more than three. The Greek letter π (pi) is used to represent this number:

$$\pi = \frac{C}{d} \approx 3.14159265358979....$$

a. The relationship between circumference and diameter is often written as $\pi \cdot d = C$. Is this the same as writing $\frac{C}{d} = \pi$? Discuss this question with your team, and be prepared to share your **reasoning** with the class.

b. If the diameter of a circle is 100 inches, what is the circumference?

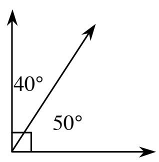
c. If the circumference of a circle measures 96 feet, what is the diameter?

METHODS AND MEANINGS

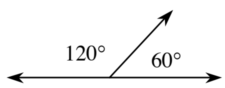
MATH NOTES

Angle Relationships

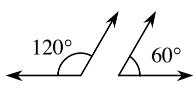
If two angles have measures whose sum is 90°, they are called **complementary angles**. For example, in the diagram at right, the 40° and 50° angles are complementary because together they form a right angle.



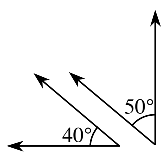
If two angles have measures whose sum is 180°, they are called **supplementary angles**. For example, in the diagram at right, the 60° and the 120° angles are supplementary because together they form a straight angle.



Two angles do not have to share a vertex to be complementary or supplementary. The first pair of angles at right are supplementary; the second pair of angles are complementary.



Supplementary

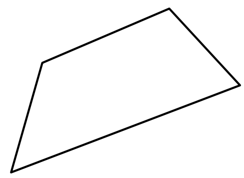


Complementary

5-110. Remember that the circumference of a circle is equal to π times the diameter. Find the circumference of a circle with:

a. diameter = 10 b. radius = 10 c. diameter = 15

5-111. Alexa is looking at the shape at right. She needs to make several different figures that are similar to it.



a. If she uses the scale factor $\frac{8}{5}$, will the new shape that she creates be larger or smaller than the original? **Justify** your answer.

b. List two different scale factors Alexa could use to make a smaller shape.

c. List two different scale factors Alexa could use to make a larger shape.

5-112. Simplify each numerical expression.

a. $\frac{1}{2}(5+13)-4\cdot 5$ b. $(5+11)-(24-15)\cdot(3)$ c. $6^2+3\cdot 7-9\div 3$

5-113. Simplify the following variable expressions.

a. $2x+5+x-6+3x$ b. $x-8+x-5+x+1$

5-114. Rewrite each fraction below as an equivalent fraction, a decimal, and a percent.

a. $\frac{6}{18}$ b. $\frac{7}{20}$ c. $\frac{9}{10}$ d. $\frac{4}{25}$

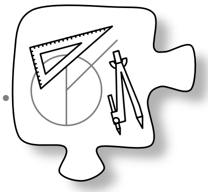
5-115. A radio station is giving away free t-shirts to students in local schools. It plans to give away 40 shirts at Big Sky Middle School and 75 shirts at High Peaks High School. Three hundred and fifty students attend Big Sky Middle School and 800 students attend High Peaks High School.

a. What is the probability of getting a t-shirt if you are a student at the middle school?

b. What is the probability of getting a t-shirt if you are a student at the high school?

c. Are you more likely to get a t-shirt if you are a student at the high school, or at the middle school?

5.3.4 How can I measure the area?



Circle Area

The ratio known as π (read, "pī") was first discovered by the Babylonians nearly 4000 years ago. Over the years, Egyptian, Chinese, and Greek mathematicians also found the constant ratio between the circumference and diameter of a circle by using measurement. The Greek letter π has been used to represent this ratio since the 1700s when it was made popular by the Swiss mathematician Euler (pronounced "oy-ler"). Even though this ratio has been known for many years, the value commonly used for π is still only an approximation.

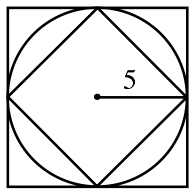
5-116. ESTIMATING CIRCLE AREA

A circle is different from any shape we have studied so far because it has no sides or corners. This makes it difficult to apply the **strategy** of decomposing it into rectangles and triangles. In this situation, estimating its area can be a useful technique. Obtain a Lesson 5.3.4A Resource Page from your teacher.

a. Using the first circle with a radius of 5 units on the resource page, estimate the area of the circle by counting whole and part squares. When each person in your team has finished their own estimate, share your results.

b. Are all of the estimates in your team identical? How can you combine your data as a team to get a new estimate that may be more accurate?

5-117. THE INSIDE OUTSIDE AVERAGE METHOD



While it is convenient to estimate area when shapes are drawn on grid paper, often shapes are not presented in that way. How can other shapes be used to help estimate area? Look carefully at the shape at right.

a. What is the area of the larger square (also shown on the resource page)?

b. What is the area of the smaller square (also shown on the resource page)?

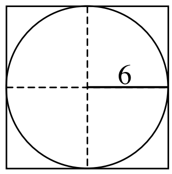
c. How does the area of each square compare to the area of the circle? Is either a good estimate of the circle's area? Why?

d. Discuss with your team how to use the areas of both the squares to estimate the area of the circle. You might consider using what you have learned about analyzing data. Explain your thinking.

5-118. Julia needs to estimate the area of a circle with a radius of 6. She is planning to use the Inside Outside Average method to make her estimate.

a. On the Lesson 5.3.4A Resource Page, estimate the area of the circle with a radius of 6 by counting whole squares and parts of squares. Share your results with your team.

b. When Julia was calculating the area of the outside square, she started by dividing the large square into four smaller ones. Her picture is at right.

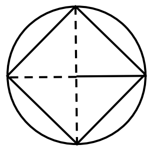


To find the area of each small square, Julia multiplied the length times the width:

$$\text{Area} = 6^2 = 36$$

How are the length, width, and area of each small square related to the radius of the circle? How is the area of the outside square related to the radius?

c. Julia divided the inside square into triangles. How is each triangle related to the radius? How is the area of the inside square related to the radius?



d. If the area of the two squares is related to the radius of the circle, Julia wonders if she can relate her circle estimate back to the radius as well. With your team, look at the estimate for area you have from part (a). How is that area estimate related to the radius of the circle?

5-119. Julia is looking at a new square with a radius of 10. So far on her paper, she has written the work at right. What expression should she write to estimate the area of the circle?

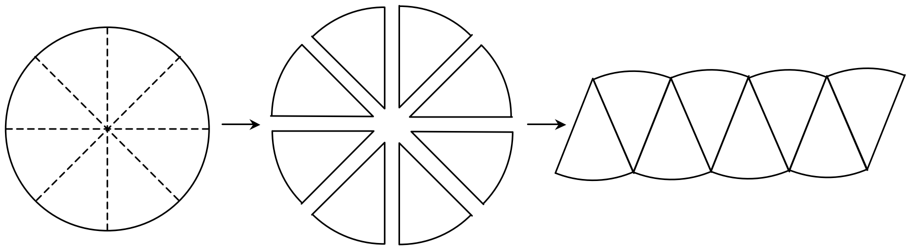
Outside area $= 4(10)^2$
Inside area $= 2(10)^2$

5-120. A FORMULA FOR CIRCLE AREA

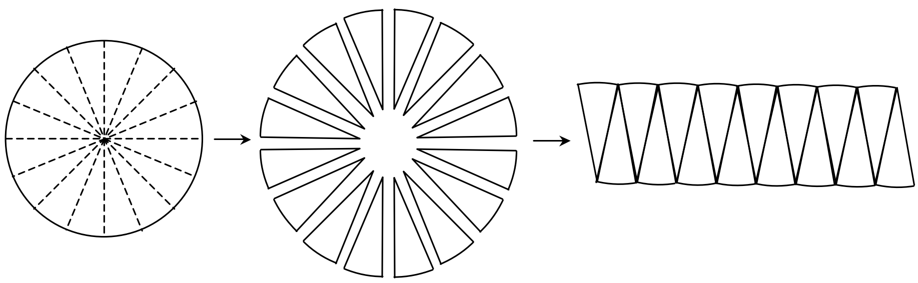
By using the Inside Outside Average method for estimating, you discovered that the area of a circle is approximately three times the radius squared. There is another way to decompose and rearrange a circle in order to find a formula for its area. As you go through this process, think about these questions:

- *If a circle is cut up and rearranged into a new shape, is the area of the new shape the same as the area of the circle?*
- *How are the dimensions (base and height) of the new shape related to the circle?*
- *Would this process work for any circle? Does it matter how large the circle is?*

Take a circle and cut it into 8 equal parts and arrange them like this:



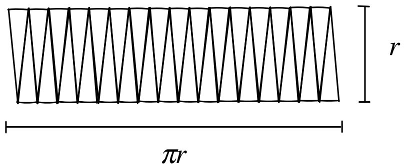
This rearrangement of the circle would look even more like a parallelogram or rectangle if we cut the circle into more pieces:



Problem continues on next page. →

5-120. *Problem continued from previous page.*

The measurements of this parallelogram are actually related to the original measurements of the circle. The height of the parallelogram is the same as the radius of the circle:



The length of the parallelogram is approximately half of the circumference of the circle. Since the circumference is equal to πd, half of the circumference is equal to:

$$\frac{\pi d}{2} = \frac{\pi(2r)}{2} = \pi r \ .$$

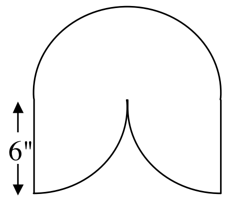
Substituting these values into the formula for the area of a parallelogram, you get:

$$A = hb = r(\pi r) = r^2\pi \ .$$

5-121. Calculate the area for the circles with a radius of 5 and 6 using this new method (the formula $A = r^2\pi$) and compare your answers to your team's estimates in problems 5-117 and 5-118. How are the areas using this new method different from the estimates in the Inside Outside Average Method?

5-122. How does the area that you found in problem 5-116 (counting estimate) compare to the calculating the area of a circle with a radius of 5 using the formula $A = r^2\pi$?

5-123. **Additional Challenge:** A circle with radius 6 inches is cut into four quarters. Those pieces are rearranged into the shape at right. Find the area and the perimeter of the shape.



5-124. LEARNING LOG

Write a Learning Log entry that explains how to use the formula for the area of a circle. Include a solved example problem. Title this entry "Area of Circles" and include today's date.

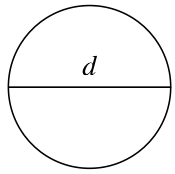
METHODS AND MEANINGS

MATH NOTES

Circumference and Area of Circles

The **circumference** (C) of a circle is its perimeter, that is, the "distance around" the circle.

$C = \pi \cdot d$



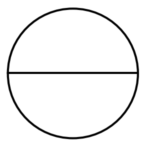
The number π (read "pi") is the ratio of the circumference of a circle to its diameter. That is, $\pi = \frac{\text{circumference}}{\text{diameter}}$. This definition is also used as a way of computing the circumference of a circle if you know the diameter as in the formula $C = \pi d$ where C is the circumference and d is the diameter. Since the diameter is twice the radius (that is, $d = 2r$) the formula for the circumference of a circle using its radius is $C = \pi(2r)$ or $C = 2\pi \cdot r$.

The first few digits of π are 3.141592.

To find the **area** (A) of a circle when given its radius (r), square the radius and multiply by π. This formula can be written as $A = r^2 \cdot \pi$. Another way the area formula is often written is $A = \pi \cdot r^2$.

Review & Preview

5-125. The circle shown at right has a diameter of 20 cm.

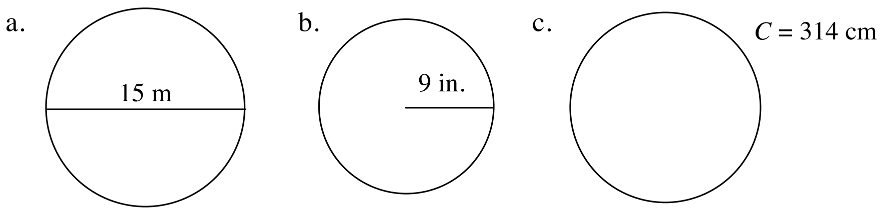


a. What measurement do you need to find in order to calculate the area inside the circle?

b. Find the area inside the circle using the formula $A = r^2\pi$. Write your answer as a product of r^2 and π, and as an approximation using $\pi \approx 3.14$.

5-126. Quintrell was writing a "Guess My Number" game. He decided to write the clues in a different way. He wrote, *"When 35 is added to my number, the answer is 4 times my original number plus 8. What is my number?"* Use the 5-D Process to find Quintrell's number.

5-127. Find the area and circumference of the following circles. You may want to refer to the Math Notes box in this lesson.

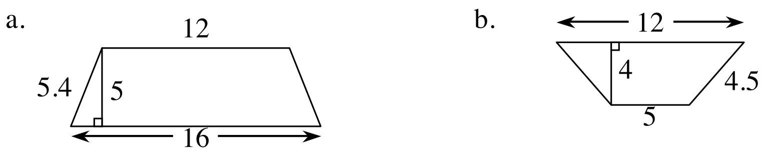


5-128. One student rewrote the expression $17 \cdot 102$ as $17(100+2)$, and then simplified to get the expression 1700+34.

a. Are the three expressions equal? **Justify** your answer.

b. What property of numbers does this demonstrate?

5-129. Find the area of each of these trapezoids. Show all of your steps.



5.4 How are they related?

Mid-Course Reflection Activities

The activities in this section review several major ideas you have studied so far. As you work, think about the topics you have studied and the activities that you have done during the first half of this course and how they connect to each other. Also think about which concepts you are comfortable using and those with which you need more practice.

As you work on this activity, keep in mind the following questions:

What mathematical concepts have you studied in this course so far?

What do you still want to know more about?

What connections did you find?

5-130. MEMORY LANE

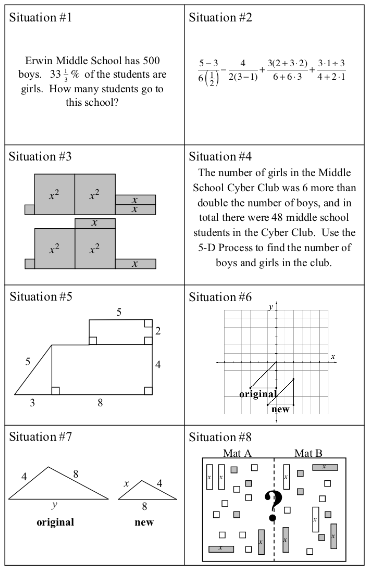
Have you ever heard someone talk about "taking a trip down memory lane"? People use this phrase to mean taking time to remember things that have happened in the past, especially things that a group of people have in common.

As you follow your teacher's directions to visit your mathematical "memory lane," think about all the activities you have done and what you have learned in this class so far this year. Your Toolkit should be a useful resource to help you with this activity.

Focus your memories in six areas:

- Probability
- Data and Statistics
- Algebraic Thinking
- Geometry
- Representing and Working with Numbers
- Graphing

5-131. SCAVENGER HUNT



Today your teacher will give you several clues about different mathematical situations. (These clues can be found on a resource page at www.cpm.org/students.) For each clue, work with your team to find all of the situations (posted around the classroom or provided on a resource page) that match each clue. Remember that more than one situation — up to five — may match each clue. Once you have decided which situation(s) match a clue, defend your decision to your teacher and receive the next clue. Be sure to record your matches on paper.

Your goal is to find the match (or more than one match) for each different clue.

5-132. WAYS OF THINKING

This course focuses on five different **Ways of Thinking**: generalizing, reasoning and justifying, reversing thinking, choosing a strategy, and visualizing. These are some of the ways in which you think while trying to make sense of a concept or while solving a problem (even outside of math class). During this course, you have probably used each Way of Thinking multiple times without even realizing it!

The following problems will focus on the Ways of Thinking. As you work them, refer to the description of the Ways of Thinking found at the end of this lesson, and think about which Ways of Thinking you are using for each problem.

1. DESIGN YOUR OWN TRICK

   In this question, you will be designing your own number trick. Use these questions to guide your thinking.

   a. If your trick includes a subtraction step, what step will likely follow later? Why?

   b. If you "multiply by five" in your trick, what inverse operation will likely follow later?

   c. It is helpful to **visualize** how the expressions are being built up and reduced back down so that the original number does not matter. Do you prefer to use variable expressions or algebra tiles when you **visualize** how a variable can represent any number? Why?

   d. Individually or with your team design a new number trick that always results in the same integer answer no matter what number you start with. Organize your trick in a table that lists the steps, include three trials to show it works with specific numbers, and include the algebra tile drawings or expressions to show it works with any number.

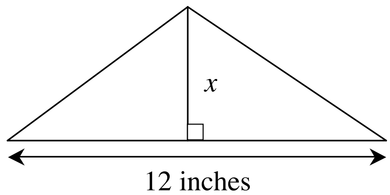
2. Use the 5-D Process and table below to find a solution. Write a possible word problem that would fit the table.

   Describe/Draw:

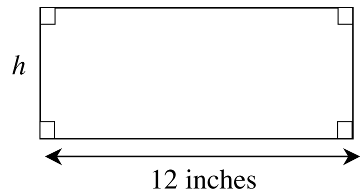
	Define and Predict		**Do**	**Decide**
	Length	Width	Perimeter	Target Perimeter = 88 ft
Trial 1:	10	$2 \cdot 10 + 5 = 25$	$10 + 25 + 10 + 25 = 70$	70 Too small

3. Each of the following shapes is missing either the height and/or the perimeter. Find the missing value for each problem. Explain your **strategy** for solving each problem.

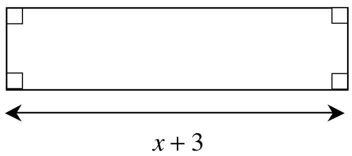
   a. Area = 24 sq. inches
   Perimeter ≈ 26.4 inches
   Height = ?



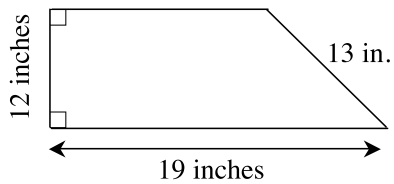
   b. Area = 42 sq. inches
   Height = ?
   Perimeter = ?



   c. Area = $4x+12$
   Perimeter = ?
   Height = ?



   d. Area = 198 sq. inches
   Perimeter = ?
   Top Base = ?



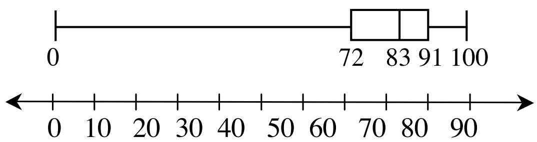
4. Design a data set that satisfies the requirement listed each time.

   a. Write six ages that would have modes of 12 and 13.

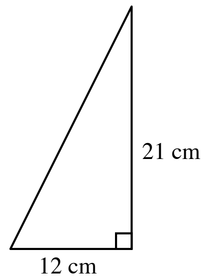
   b. Write five heights that would have a median of 63 inches.

   c. Write four different quiz scores that would have a mean of 75%.

   d. Write 11 test scores that would be represented with the following box-and-whisker plot.



5. Use the triangle at right to answer the questions below.



a. Enlarge the triangle to create a similar triangle with side lengths that are twice as long as in the diagram. Label the new triangle B and be sure to label the lengths that you can determine.

b. Reduce the original triangle to create a similar triangle with side lengths that are one-third as long as in the diagram. Label it triangle C and be sure to label the lengths that you know.

c. Find the area of each triangle.

d. How do the areas of the enlarged and reduced triangles compare to the original? Be as specific as you can.

e. If triangle B were dilated to become triangle C, what would the scale factor be? How do you know?

Ways of Thinking

Reasoning and Justifying:

To use logical reasoning means to organize information in order to draw a conclusion. When you explain why you think an idea is true, you are justifying. You often think this way when you try to convince yourself or someone else that an idea or solution is correct. Often, a justification is the answer to the questions, "Why?" or, "How do you know for sure?" When you think or say, *"I think this is true because…"*, you are justifying.

Choosing a Strategy:

To choose a strategy means to think about what you know about a problem and match that information with methods and processes for solving problems. As you develop this way of thinking you learn how to choose ways of solving problems based on given information. You think this way when you ask/answer questions like, "What strategy might work for…?" or , "How can I use this information to answer…?" When you are looking for a method to answer a question or solve a problem, you are choosing a strategy/tool.

Generalizing:

To generalize means to make a statement or conclusion, like a rule stating common properties, from a collection of evidence. You think this way when you describe patterns, because you are looking for a general statement that describes each term in the pattern. Often, a generalization is the answer to the question, "What is in common?" When you think or say, *"I think this is always true…"*, you are generalizing.

Reversing Thinking:

To reverse your thinking can be described as "thinking backward." You think this way when you want to understand a concept in a new direction. Often, it requires you to try to undo a process. When you think or say, *"What if I try to go backwards?"*, you are reversing your thinking.

Visualizing:

To visualize means to make a picture in your mind that represents a situation or description. As you develop this Way of Thinking, you learn how to turn a variety of situations into mental pictures. You think this way when you ask or answer questions like, "What does it look like when…?" or "How can I draw…?" When you wonder what something might look like and work to create an image of it, you are visualizing.

Chapter 5 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① WHAT HAVE I LEARNED?

Working the problems in this section will help you to evaluate which types of problems you feel comfortable with and which ones you need more help with.

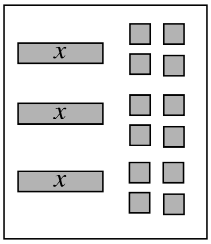
Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like them.

CL 5-133. Copy the chart on your paper. Complete two trials by reading the variable expressions. Write in the steps as well.

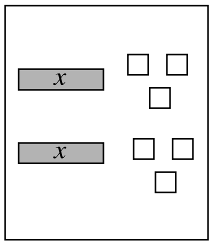
Steps	**Trial 1**	**Trial 2**	**Variable Expression**
			x
			$3x$
			$3x + 27$
			$3x + 21$
			$x + 7$
			7

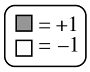
CL 5-134. When algebra tiles are grouped in sets, as shown below, they can be written in two different ways. Write two equivalent expressions that represent these collections of algebra tiles.

a.



b.

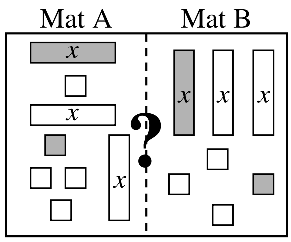




CL 5-135. Use the 5-D Process to help you solve the following problem.

Lillian and Isabella went shopping for school supplies. Lillian found her favorite notebook on sale so she bought three of the exact same notebook. Isabella also wanted a notebook. She bought one of the same notebooks that Lillian bought. Both girls started shopping with the same amount of money. If Lillian spent all of her money and Isabella had \$5 left, then how much money did each girl have when she arrived at the store?

CL 5-136. Adriel and Gustavo were playing a game with algebra tiles. As they compared them, they were left with the following expressions on their mats. Use legal moves to simplify what is on the mats. Be sure to record your work in a way that others can understand.



Can you determine which side has the greater value? If so, show why. If not, for what values of x is Mat A larger?

CL 5-137. Use graph paper to solve the following problem.

a. Draw an xy-coordinate graph and label each axis. Plot the following ordered pairs: $(3,4)$, $(3,-5)$, $(-1,-2)$, and $(-1,3)$. Connect the points in the order given as you plot them, then connect the fourth point to the first one.

b. Describe your shape. Where do you see the base(s) and height of your shape?

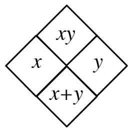
c. What is the area of your shape?

CL 5-138. Alejandra has been practicing her free-throw shots as she gets ready for basketball season. At her last practice, she made 70% of her shots from the free-throw line. If she shot the ball 130 times,

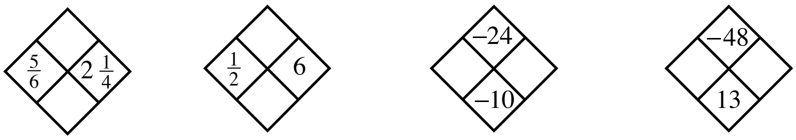
a. How many times did she make a free-throw?

b. How many times did she miss? What percentage of her shots did she miss?

CL 5-139. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



a. b. c. d.



CL 5-140. Build each collection of tiles represented below on a mat. Name the collection using a simpler algebraic expression, if possible. If it is not possible to simplify the expression, explain why not.

a. $(-x)+5-4x+x-(-3)+(-3x)$

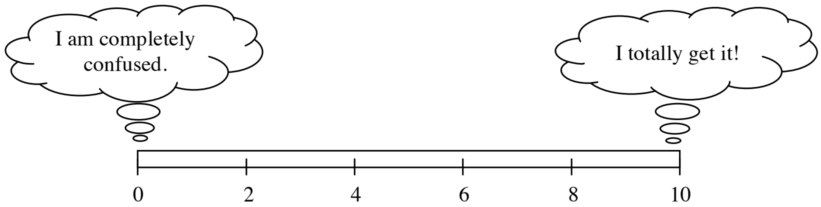
b. Nine plus four times a number, plus three minus seven times the number

c. $3-7x^2+9x$

d. $2x+3x^2-7+(-x^2)$

CL 5-141. For each of the problems in this section of closure, do the following:

a. Draw a bar or number line like the one below that represents 0 to 10.



b. Color or shade in a portion of the bar that represents your current level of understanding and comfort with completing that problem on your own.

c. If any of your bars are less than a 5, choose *one* of those problems and do one of the following tasks:

- Write two questions that you would like to ask about that problem.
- Brainstorm two things that you DO know about that type of problem.

d. If all of your bars are a 5 or above, choose one of those problems and do one of these tasks:

- Write two questions you might ask or hints you might give to a student who was stuck on the problem.
- Make a new problem that is similar and more challenging than that problem and solve it.

② WHAT TOOLS CAN I USE?

You have several tools and references available to help support your learning – your teacher, your study team, your math book, and your Toolkit to name only a few. At the end of each chapter you will have an opportunity to review your Toolkit for completeness as well as to revise or update it to better reflect your current understanding of big ideas.

The main elements of your Toolkit should be your Learning Log, Math Notes, and the vocabulary used in this chapter. Math words that are new to this chapter appear in bold in the text. Refer to the lists provided below and follow your teacher's instructions to revise your Toolkit, which will help make it useful for you as you complete this chapter and as you work in future chapters.

Learning Log Entries

- Lesson 5.1.3 – Simplifying Expressions
- Lesson 5.2.2 – Simplifying Expressions (Legal Moves)
- Lesson 5.2.4 – Finding Boundary Points
- Lesson 5.3.4 – Area of Circles

Math Notes

- Lesson 5.1.2 – Algebraic Expressions
- Lesson 5.1.3 – Distributive Property
- Lesson 5.2.1 – Inequality Symbols
- Lesson 5.2.2 – Additive Identity and Additive Inverse
- Lesson 5.3.1 – Angles and Their Measures
- Lesson 5.3.2 – Circle Vocabulary
- Lesson 5.3.3 – Angle Relationships
- Lesson 5.3.4 – Circumference and Area of Circles

Mathematical Vocabulary

The following is a list of vocabulary found in this chapter. The words in bold are the words new to this chapter. It is a good idea to make sure that you are familiar with these words and know what they mean. For the words you do not know, refer to the glossary or index. You might also want to add these words to your Toolkit for a way to reference them in the future.

acute angle	**boundary point**	**circumference**
complementary angle	**diameter**	**Distributive Property**
equilateral triangles	equivalent expressions	**inequality**
line segment	**midpoint**	obtuse angle
parallel	perpendicular	**perpendicular bisector**
pi (π)	**radius**	right angle
supplementary angle		

Process Words

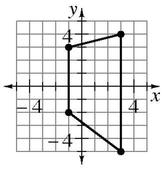
These words describe problem solving strategies and processes that you have been involved in as you worked in this chapter. Make sure you know what each of these words means. If you are not sure, you can talk with your teacher or other students or look through your book for problems in which you were asked to do these things.

analyze	construct	estimate
explain	express	generate
justify	organize	record
represent	reverse	simplify expressions

Answers and Support for Closure Activity #1

What Have I Learned?

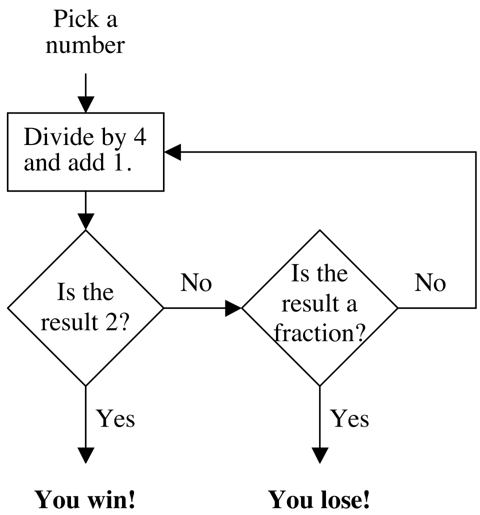
Problem	Solution	Need Help?	More Practice
CL 5-133.	Pick a number Multiply by 3 Add 27 Subtract 6 Divide by 3 Subtract the original number.	Lessons 5.1.1 and 5.1.2 Math Notes box in Lesson 5.1.2	Problems 5-2 through 5-4, 5-6, 5-9, 5-14, and 5-20
CL 5-134.	a. $3(x+4)$ and $3x+12$ b. $2(x-3)$ and $2x-6$	Lessons 5.1.2 and 5.1.3 Math Notes box in Lesson 5.1.3 Learning Log (problem 5-33)	Problems 5-15, 5-19, 5-29, and 5-45
CL 5-135.	Each girl started with \$7.50.	Lessons 3.2.2 through 3.2.5 Math Notes box in Lesson 3.2.3 Learning Log (problem 3-98)	Problems CL 3-113, CL 4-113, 5-93, 5-102, and 5-126
CL 5-136.	Mat A has -3 and Mat B has -2, so Mat B is greater.	Lessons 5.2.1 and 5.2.2 Math Notes box in Lesson 5.2.1 Learning Log (problem 5-54)	Problems 5-40 through 5-44, 5-50 through 5-53, 5-61, 5-62, 5-63, and 5-67

Problem	Solution	Need Help?	More Practice
CL 5-137.	a.  b. There is a trapezoid with two vertical bases: $(3,4)$ to $(3,–5)$ and $(–1,–2)$ to $(–1,3)$. The height can be seen on any horizontal line segment between $x = 3$ and $x = –1$. c. 28 square units	Lesson 2.3.5 Math Notes box in Lesson 2.2.2 Learning Log (problem 2-150)	Problems 2-135, 2-137, 2-151, 4-21, 5-24, 5-57, and 5-129
CL 5-138.	a. 91 b. 39 shots, 30%	Lessons 4.1.1 and 4.1.2 Math Notes box in Lesson 4.1.2	Problems CL 4-112, 5-37, 5-58, and 5-104
CL 5-139.	a. $\frac{15}{8}$ and $\frac{37}{12}$ b. 3 and $\frac{13}{2}$ c. –12 and 2 d. 16 and –3	Lessons 1.2.4, 1.2.5, and 2.3.1 Math Notes box in Lesson 1.2.4 Learning Log (problem 1-107)	Problems CL 4-116, and 5-81
CL 5-140.	a. $-7x+8$ b. $-3x+12$ c. Fully simplified. d. $2x^2+2x-7$	Lessons 3.1.3 and 5.1.2 Math Notes boxes in Lessons 3.1.3 and 5.1.2 Learning Log (problem 3-28)	Problems CL 4-117, 5-31, 5-3, 5-46, and 5-113

Puzzle Investigator Problems

PI-9. WHAT'S MY NUMBER?

Francesca has a game for you. She decided to show the rules of the game using a flowchart, at right.



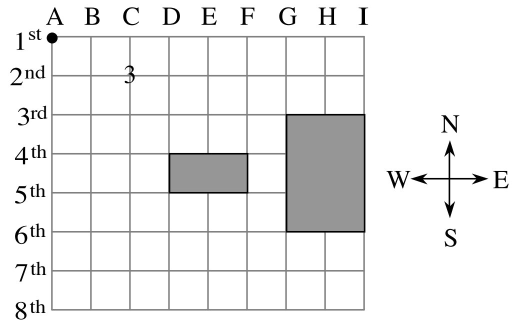
a. Show that if you start with the number 60, you will lose.

b. How many starting numbers less than 100 will win? List the possible winning numbers and show how you know they will win.

c. What if Francesca changed the rules so that you win if you end up with the number 1? What numbers would win?

PI-10. WAY TO GO!

The map at right shows the streets in Old Town. Assume Jacqueline is standing at the corner of A and 1st Streets. Assume Jacqueline will only walk South or East. The shaded rectangles represent large buildings. Assume Jacqueline will not pass through any buildings.



a. The number "3" at the intersection of C and 2nd Streets means that there are three different ways she can get there from her starting position. What are those three ways? Describe them in words.

b. How many different ways can she walk to the corner of F and 4th Streets?

c. How many different ways can she walk to the corner of D and 5th Streets?

d. Explain how you can use your answers to parts (b) and (c) to find the number of ways she can walk to the corner of F and 5th Streets. Why does this make sense?

e. Find the number of different ways she can walk to the corner of I and 8th Streets.

f. How could you change the map so that Jacqueline has only 7 ways to get to the corner of D and 3rd streets? You can remove blocks or add them.